MATH 80430, TOPICS IN TOPOLOGY FALL 2013
INTRODUCTION TO MORSE THEORY

- **Instructor**: Liviu I. Nicolaescu, 224 Hayes-Healey.
- **Schedule**: Mon-Wed-Fri: 2:00-2:50 pm 116 Pasquerilla.

During this class I intend to present some basic facts and applications of an extremely versatile topological technique known as *Morse theory*. The main idea behind this technique can be easily visualized.

Suppose that M is a smooth, compact manifold which for simplicity we assume is embedded in an Euclidean space E. We would like to understand basic topological invariants of M such as its homology and we attempt a "slicing" technique. We fix a unit vector $\vec{u}$ in E and we start slicing M with the family of hyperplanes perpendicular to $\vec{u}$. Such a hyperplane will in general intersect M along a submanifold (slice). The manifold can be recovered by continuously stacking the slices on top of each other in the same order as they were cut out of M.

The main idea can be visualized as follows. Think of the collection of slices as a deck of cards of various shapes. If we let these slices continuously pile up in the order they were produced we notice and increasing stack of slices. As this stack grows we notice that there are moments of time when its shape suffers a qualitative change. Morse theory is about extracting quantifiable information by studying the evolution of the shape of this growing stack of slices.

Making this idea work in concrete applications requires quite a bit of ingenuity and a rather detailed understanding of the geometry of the situation.

I will devote about a third of the class developing the foundations of this theory, while the remaining, meatier, part is devoted to some of its varied applications: the surgery description of 3-manifolds, the topology of Grassmannians, the topology of robot arm motions, moments maps and their topology.

I will follow rather closely my book [1], and I plan to cover most of the the first three chapters in the book. This book is freely available on-line through the Math Library. There will be three homework assignments, 4-5 problems each. This class assumes familiarity with the basics of calculus on smooth manifolds, and working knowledge of singular (co)homology.

REFERENCES

[1] L.I. Nicolaescu: *An Invitation to Morse Theory*, 2nd Edition, Springer Verlag 201.