MATH 80220, TOPICS in ALGEBRA II, Spring 2014
Introduction to Algebraic Number Theory

Modern number theory lies at the interplay between algebra, geometry, representation theory and analysis and the interactions among these fields led to numerous advances in the last half century. This course is an introduction to algebraic number theory from a computational perspective: we will study the main objects (number fields and their rings of integers), their properties from a classical perspective, and their connections to algebra, geometry and analysis, while at the same time interact computationally with the main theorems of algebraic number theory in the open source computational algebra system Sage.

The main topics of the course will include: number fields and number rings; the class group and its structure using Minkowski's geometry of numbers; Kummer's special case of Fermat's Last Theorem; decomposition groups and ramification; the Dirichlet unit theorem; zeta functions and the class number formula (a special case of the million dollar Birch and Swinnerton-Dyer conjecture); L-functions and primes in arithmetic progression. Each topic will be accompanied by ample experimentation in Sage. Knowledge of rings and fields is required, while Galois theory and complex analysis are useful but not prerequisites.