MATH 80870: Topics in Mathematical Physics: Positive Grassmannians and Applications
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The discovery of cluster algebras by Fomin and Zelevinsky was in large part motivated by the study of the phenomenon of total positivity in reductive Lie groups and their homogeneous spaces. In a Grassmannian of k-dimensional planes in an n-dimensional space, a totally nonnegative element is represented by a k x n matrix of full rank with all k x k minors nonnegative.

Grassmannians were among the first examples of varieties that support a natural cluster structure, that was arrived at in two ways: combinatorial, using Postnikov's work on parametrizing totally nonnegative cells in Grassmannians using directed planar networks, and geometric, that utilized properties of Grassmannians as Poisson homogeneous spaces. The course will start with an overview of these two approaches and of combinatorial and algebraic structures that arise in the process. We will then turn to applications, including an unexpected relation between positive Grassmannians and soliton solutions of the Kadomtsev-Petviashivili (KP) partial differential equation which (among other things) describes waves in shallow water. Finally, time permitting, we will consider a new setting in which totally positive Grassmannians have appeared recently - the computation of scattering amplitudes in the planar limit of maximally extended supersymmetric Yang-Mills theory (SSYM).

After traditional lectures in the beginning, much of the course will be based on student presentations. The time of the meetings will be chosen to fit schedules of interested students. A list of useful references includes:

1. A. Postnikov, Total positivity, Grassmannians, and networks, http://arxiv.org/pdf/math/0609764v1.pdf

2. M. Gekhtman, M. Shapiro and A. Vainshtein, Cluster algebras and Poisson geometry, AMS Mathematical Surveys and Monographs, volume 167 (2010), chapters 2, 4, 8.

3. Y. Kodama, L. Williams, KP solitons and total positivity for the Grassmannian, Invent. Math. 198 (2014), pp. 637-699.

4. S. Franco, D. Galloni and A. Mariotti, Bipartite field theories, cluster algebras and the Grassmannian, J. Phys. A 47 (2014), http://iopscience.iop.org/1751-8121/47/47/474004/article;jsessionid=8459489A55EC464D3ABC6B89B3FA755A.c2