# TOPICS IN DIFFERENTIAL GEOMETRY: RIGIDITY PROBLEMS
80750 / Spring 2016 / Karsten Grove

A genuine *Rigidity Theorem* asserts that a space subject to suitable restrictions is isometric to a given one (or a list of spaces), in particular there are no local deformations subject to the given restrictions.

In contrast to the rich Teichmüller Theory of hyperbolic structures on 2-manifolds, Mostow's famous rigidity theorem asserts that in dimensions at least 3, a finite volume hyperbolic manifold is determined up to isometry by its fundamental group. In variable curvature, there are by now several s-called *rank rigidity theorems*, asserting that in higher (geometric) ranks its universal cover either splits isometrically or is isometric to a symmetric space.

The plan of the course is to primarily focus on the so-called *Blaschke conjecture*, which goes back to the 1930's. In its modern formulations it states that

*A Riemannian manifold with the same injectivity radius and diameter is a compact rank one symmetric space*

Here, the *injectivity radius* of a manifold the largest $r$ such that the exponential map restricted to the open $r$ ball at any tangent space is a diffeomorphism onto its image.

It is known that any Blaschke manifold (injectivity radius = diameter) has all geodesics closed of the same length and has the cohomology of a rank one symmetric space. Moreover, in the "spherical case" the conjecture has been proved by M. Berger using a crucial estimate due to J. Kazdan. Other partial results a re known, but there are also mistakes in the literature.

The mathematics needed and developed for this problem is both beautiful, diverse and useful in other contexts as well.

*Prerequisits:* Basic Riemannian Geometry corresponding for example to MATH 60670 will be good.

## References

The main sources for the course (other than research articles) are given below


- *Manifolds all of whose Geodesics are Closed*, A. L. Besse, Ergebnisse der Mathematik 93, Springer-Verlag, 1978
- *Lectures on the Blaschke Conjecture*, W. Ballmann, 2014, http://people.mpim-bonn.mpg.de/hwbllmnn/notes.html