

Fall 2016
MATH 80430
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Topics in topology: Batalin-Vilkovisky formalism and applications in topological quantum field theory.

The Batalin-Vilkovisky (“BV”) formalism arose in the end of 1970’s/beginning of 1980’s as a tool of mathematical physics designed to define the path integral for gauge theories. Since then the construction turned out to be very useful for applications in algebraic topology – invariants of 3-manifolds and knots, Chas-Sullivan string topology, operations on rational cohomology of CW complexes. Another spectacular application of the BV formalism is Kontsevich’s deformation quantization of Poisson manifolds. The general direction these applications go is via applying the BV formalism to define the path integral for particular models of topological field theory and then finding an appropriate interpretation for the value of the path integral (and proving the desired properties).

The goal of the course is to provide an introduction to the Batalin-Vilkovisky formalism and some of its applications.

The course is intended to be self-contained; no background in mathematical physics is assumed.

Tentative program:

- Examples of classical gauge theories: Chern-Simons theory, Yang-Mills theory. Ref: [Reshetikhin].
- Perturbed (finite-dimensional) Gaussian integrals and Feynman diagrams. Ref: [Etingof, Reshetikhin].
- Batalin-Vilkovisky formalism: odd-symplectic geometry, integration theory on supermanifolds. Ref: [Schwarz92, AKSZ95, Losev].
 - BV as a solution to the “gauge-fixing problem”.
- Example: a topological quantum field theory (“non-abelian BF theory”) on CW complexes. Ref: [Mnev06, Mnev08, CMR cell].
 - Fiber BV integral as homotopy transfer of algebraic structures.
 - Relation to formal geometry of the moduli space of local systems and to behavior of R -torsion in the neighborhood of singularities on the moduli space.
- Perturbative Chern-Simons theory: applications to invariants of 3-manifolds and knots. Ref: [AS91, AS93, Kontsevich92, CM08].
 - Feynman diagrams as integrals over Fulton-MacPherson-Axelrod-Singer compactified configuration spaces of points.
- Poisson sigma model on a 2-disk and Kontsevich’s deformation quantization of Poisson manifolds. Ref: [CF99].
- BV formalism + Segal’s gluing/cutting axiom = BV-BFV formalism. Ref: [CMR12, CMR15]; short survey in [CMR survey].
 - Combinatorial example: non-abelian BF theory on cobordisms with CW decomposition. Ref: [CMR cell].
 - AKSZ theories on cobordisms via configuration space integrals. Ref: [CMR15].

Some references:

Lecture notes and surveys:

- [Etingof] P. Etingof, *Mathematical ideas and notions of quantum field theory*, <http://www-math.mit.edu/etingof/lect.ps> (2002).
- [Losev] A. Losev, *From Berezin integral to Batalin-Vilkovisky formalism: a mathematical physicist's point of view*, <http://homepages.spa.umn.edu/shifman/Berezin/Losev/Losev.pdf> (2007).
- [Reshetikhin] N. Reshetikhin, *Lectures on quantization of gauge systems*, arXiv:1008.1411.
- [CMR survey] A. S. Cattaneo, P. Mnev, N. Reshetikhin, *Perturbative BV theories with Segal-like gluing*, arXiv:1602.00741.

Papers:

- [Schwarz92] A. S. Schwarz, *Geometry of Batalin-Vilkovisky quantization*, arXiv:hep-th/9205088, *Comm. Math. Phys.* 155 (1993) 249–260.
- [AKSZ95] M. Alexandrov, M. Kontsevich, A. S. Schwarz, O. Zaboronsky, *The geometry of the master equation and topological quantum field theory*, arXiv:hep-th/9502010, *Int. J. Mod. Phys. A* 12.07 (1997) 1405–1429.
- [AS91] S. Axelrod, I. M. Singer, *Chern-Simons Perturbation Theory*, arXiv:hep-th/9110056.
- [AS93] S. Axelrod, I. M. Singer, *Chern-Simons Perturbation Theory II*, arXiv:hep-th/9304087, *J. Diff. Geom.* 39 (1994) 173–213.
- [Kontsevich92] M. Kontsevich, *Feynman diagrams and low-dimensional topology*, First European Congress of Mathematics Paris, July 6–10, 1992. Birkhäuser Basel, 1994, [http://www.ihes.fr/maxim/TEXTS/Feynman diagrams and low-dimensional topology.pdf](http://www.ihes.fr/maxim/TEXTS/Feynman%20diagrams%20and%20low-dimensional%20topology.pdf)
- [CF99] A. S. Cattaneo, G. Felder, *A path integral approach to the Kontsevich quantization formula*, arXiv:math/9902090, *Comm. Math. Phys.* 212 (2000) 591–611.
- [CM08] A. S. Cattaneo, P. Mnev, *Remarks on Chern-Simons invariants*, arXiv:0811.2045, *Comm. Math. Phys.* 293 (2010) 803–836.
- [CMR12] A. S. Cattaneo, P. Mnev, N. Reshetikhin, *Classical BV theories on manifolds with boundary*, arXiv:1201.0290, *Comm. Math. Phys.* 332.2 (2014) 535–603.
- [CMR15] A. S. Cattaneo, P. Mnev, N. Reshetikhin, *Perturbative quantum gauge theories on manifolds with boundary*, arXiv:1507.01221.
- [Mnev06] P. Mnev, *Notes on simplicial BF theory*, arXiv:hep-th/0610326, *Moscow Math. J.* 9.2 (2009) 371–410.
- [Mnev08] P. Mnev, *Discrete BF theory*, arXiv:0809.1160, Ph.D. dissertation.
- [CMR cell] A. S. Cattaneo, P. Mnev, N. Reshetikhin, *Cellular BV-BFV-BF theory*, in preparation.