Math 80870 - Topics in Mathematical Physics
Hayes Healy Center 125, MWF 10:30-11:20 (CRN 29559), Spring 2018

Instructor: Dylan Rupel
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Course Description: The primary goal of this course is to explain the role of quiver representations in the theory of quantum groups. I will begin the course with the basic structure theory for representations of an acyclic quiver. These foundations will allow us to construct a quantum group using the Hall-Ringel algebra of this quiver. By reinterpreting this construction geometrically we will find a remarkable basis for the quantum group and discuss the implications of these constructions on the representation theory of the corresponding Lie algebra. Along the way, we will interpret many seemingly ad-hoc comuptations performed inside the quantum group in terms of somewhat more natural constructions involving the representation theory of quivers. Depending on the timing of the course, additional topics may include: crystal bases, cluster algebras, categorification, and quantum knot invariants.

Prerequisites: The main tool in the course will be simple linear algebra, afterall quiver representations are simply collections of vector spaces and linear maps between them. Some knowledge of algebraic geometry will be needed later in the course, but to the extent it is possible I will introduce most of the necessary ideas.

Possible References:

- *Representation Theory of Artin Algebras*, Auslander, Reiten, Smalo
- *Tame Algebras and Integral Quadratic Forms*, Ringel
- *Quiver Representations*, Schiffler
- *Ringel-Hall Algebras*, Hubery (lecture notes)
- *Lectures on Hall Algebras*, Schiffmann (lecture notes)
- *Introduction to Quantum Groups*, Lusztig
- *Lectures on Quantum Groups*, Jantzen
- *Quantum Groups*, Kassel