Math 80440 Spring 2021

Topics in Topology: factorization algebras

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A *factorization algebra* on a topological space M is a structure with formal similarities to a *sheaf* on M. Like sheaves, the language of factorization algebras is versatile and shows up in quite different contexts; for example, the observables of a classical or quantum field theory can be packaged as a factorization algebra on the space-time manifold M on which the field theory lives. Dealing with factorization algebras on manifolds of dimension 1 basically amounts to homological algebra. Algebras over the little n-disk operad E_n yield examples of factorization algebras on $\mathbb{R}^n$. Factorization algebras on $\mathbb{R}^2$ with additional properties give rise to objects called *vertex algebras* which are intricate algebraic structures that have been developed to describe what physicists call *conformal field theories*.

The goal of the course is to define factorization algebras, discuss interesting examples, and to relate them to the other structures mentioned above. Along the way, we'll talk about classical field theories, which we'll use to motivate the definition of a factorization algebra. There will be some homological algebra (e.g., derived functors), some category theory (e.g., symmetric monoidal categories, (homotopy) colimits, Kan extensions) and some physics.

The only prerequisite for the course is standard first year graduate material (e.g., manifolds, vector bundles, differential forms, chain complexes, homology and categories) and a willingness to learn new stuff. In particular, no background in physics is assumed. All background material in physics, geometry, topology and category theory will be covered as needed.

The course will be loosely based on the book entitled *Factorization algebras in quantum field theory* by Kevin Costello and Owen Gwilliam, available at https://people.math.umass.edu/~gwilliam/vol1may8.pdf, as well as incomplete course notes of a previous classes I taught on this topic, in the Spring of 2014 https://www3.nd.edu/~stolz/Math80440(S2014)/Fact_algs.pdf.