TOPICS IN ALGEBRA, 80210, FALL 2022, MW 2-3:15, HH 125

ALGEBRAIC D-MODULES, Sam Evens

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Algebraic D-modules are modules over algebraic differential operators on a smooth variety. We will begin by studying modules over the Weyl algebra A_n, which is the algebra of polynomial differential operators on $\mathbf{C}^n$. Later we will study differential operators on a general smooth variety and their modules, which will require some of the language of sheaves, which I'll introduce when needed. We will show how to use geometric techniques to construct D-modules, and discuss notions like holonomic modules, the characteristic variety, functoriality, and the construction of irreducible holonomic D-modules using the intermediate extension. Time permitting and depending on the interests of the audience, we will discuss regular singularities, the Riemann-Hilbert correspondence between D-modules and constructible sheaves, and/or applications to representation theory. I'll give some exercises which we will discuss in class.

PREREQUISITES: In principle, this course should be understandable to a student who has completed the first year of graduate algebra (60210-60220), but the class will be easier to understand for students who have had some introduction to algebraic geometry.

The best sources are lecture notes, including:

(1) Lecture notes by Victor Ginzburg
https://people.math.harvard.edu/ gaitsgde/grad_2009/Ginzburg.pdf
(2) Lecture notes by Braverman, Chmutova, Etingof, and Yang
https://math.mit.edu/ xxu/Course/18.748/EtingofLec.pdf

Some useful books are:

(3) J. Bjork, Analytic D-modules and applications, Springer 1993
(4) A. Borel, P.-P. Grivel, B. Kaup, A. Haefliger, B. Malgrange, F. Ehlers, F, Algebraic D-modules, Perspectives in Mathematics, 2. Academic Press, Inc., Boston, MA, 1987
(5) S. Coutinho, A Primer of Algebraic D-modules, Cambridge University Press, 2009.