

Fall 2022, MATH 80510, Topics in Mathematical Logic, MWF 9:25 – 10:15.

Ultrafilters and Ramsey Theory

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Ultrafilters and Ramsey theory work symbiotically: ultrafilters are used to prove various Ramsey theorems, while Ramsey theory is used to solve structural problems involving ultrafilters. In this course, we will investigate two archetypes (with many variations) of Ramsey theory and ultrafilters working together. After a quick review of ordinals and cardinals, we will introduce the customary special types of ultrafilters such as Ramsey ultrafilters and p -points, and go through key construction methods from the Continuum Hypothesis, Martin's Axiom, and forcing. After this, the first main theme will be the interrelations between topological Ramsey spaces and ultrafilters. The second main theme will be ultrafilter methods used in proofs of Hindman's Theorem and the Hales-Jewett Theorem. No prior knowledge of forcing is required for this course, but mathematical maturity gained from previous graduate courses is required. Requests for more material involving forcing will be happily met.

Main Course Texts:

Set Theory, by Kenneth Kunen, 2013.

Introduction to Ramsey Spaces, by Stevo Todorćević, 2010.

Supplemental Material will be taken from the following:

Ramsey Methods in Analysis, by Spiros Argyros and Stevo Todorćević, 2005.

Set Theory: On the Structure of the Real Line, by Tomek Bartoszyński and Haim Judah, 1995.

Related research articles will be used throughout.