Spring 2023 MATH 80870

Topics in Math. Physics: Total Positivity and Applications

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MWF 12:50 - 1:40 pm

A matrix is called totally positive (TP) if all its minors are positive. TP matrices were first studied in classical works of Schoenberg and Gantmakher-Krein in 1930s and since then found numerous applications in analysis, probability, combinatorics and, more recently, in representation theory and integrable systems. The notion of total positivity was extended to reductive Lie groups by Lusztig and served as a main motivation to Fomin and Zelevinsky in their discovery of cluster algebras. We will start with classical results on spectral properties of TP matrices and TP integral kernels, discuss connections with Polya frequency sequences and then parameterizations of totally positive loci and tests for total positivity. We will then turn to total positivity in reductive Lie groups with applications to integrable systems including Toda flows. Then we will turn to positivity in the context of cluster algebras using TP Grassmannians as the main example.

Some of the sources we will use are listed below.

References

[1] S. M. Fallat and C.R. Johnson, Totally nonnegative matrices, Princeton University Press, 2011.

[2] S. Fomin, L. Williams and A. Zelevinsky, Introduction to Cluster Algebras, arXiv:1608.05735, arXiv:2008.09189, arXiv:2106.02160.

[3] S. Fomin and A. Zelevinsky, Total positivity: tests and parametrizations, Math. Intelligencer 22 (2000), 23-33.

[4] S. Fomin and A. Zelevinsky, Double Bruhat cells and total positivity, J. Amer. Math. Soc. 12 (1999), 335-380.

[5] S. Karlin, Total positivity., Vol. I, Stanford University Press, Stanford, CA, 1968.

[6] G. Lusztig, Introduction to total positivity, de Gruyter Exp. Math. 26 (1998), 133–145.

[7] A. Postnikov, Total positivity, Grassmannians, and networks, arXiv:math/0609764.