Math 80350, Topics in Analysis – Spring 2023

MWF 10:30-11:20, HAYE 125

Alex Himonas

Description: In this Partial Differential Equations (PDE) topics course we begin with background material from Fourier Analysis and Distributions. Then, we will introduce Sobolev spaces and discuss their main properties including the Sobolev Imbedding Theorem and the Algebra property. From the linear PDE we will discuss the Laplace, Heat, and Wave equations as models of linear elliptic, parabolic, and hyperbolic equations. Also, we will discuss the linear Schrödinger and the linear KdV equations. From the nonlinear PDE we will consider the Korteweg-de Vries (KdV) equation and the nonlinear Schrödinger equation (NLS) as the basic models of dispersive equations describing water waves and nonlinear optics, among many other physical situations. For these we will present traveling wave solutions (solitons) and will discuss their initial and boundary value problems. Also, we will discuss a diffusion equation with variable coefficients that arises in finance and economics as an asset pricing model.

Our main objective is to introduce the students to some aspects of the basic theory of PDE. No prior knowledge in PDE is assumed. However, it is expected that students are familiar with basic Real Analysis (Lebesgue integration and differentiation). This course should be of interest to graduate students in Mathematics, Sciences, Engineering, Finance and Economics.

Prerequisites: Basic Analysis I. Some familiarity with Ordinary Differential Equations is desirable but not a prerequisite. Exposure to an undergraduate PDE course is helpful but not required.

References

[B] J. Bourgain, *Global Solutions of Nonlinear Schrödinger Equations*, AMS, (1999), ISBN 0-8218-1919-4.

[E] L. Evans, *Partial Differential Equations*, AMS, Graduate Studies in Mathematics, **19**, (1998), ISBN 0-8218-0772-2. (*The standard graduate textbook in PDE.*)

[F] A.S. Fokas, *A Unified Approach to Boundary Value Problems*, CBMS **78**, SIAM (2008), ISBN 978-0-898716-51-1.

[Fo] G.B. Folland, *Introduction to Partial Differential Equations*, Princeton University Press, (1995), ISBN 0-691-04361-2.

[Hi] A. Himonas, *Lecture Notes in Partial Differential Equations*, University of Notre Dame, (2023).

[H] L. Hörmander, *The Analysis of Linear Partial Differential Equations*, Vol. I, Springer, (1983), ISBN 0-540-52345-6. (*A good book for Fourier Analysis and Distributions.*)

[J] F. John, *Partial Differential Equations*, Springer, (1982). (*A classic developping the basic theory of PDE, linear and nonlinear, concisely.*)

[LP] F. Linares and G. Ponce, *Introduction to nonlinear dispersive equations*, Springer Verlag. (*A good book on dispersive equations, like the KdV and NLS.*)

[S] W. Strauss, *Partial Differential Equations: An Introduction*, John Wiley & sons, 2008. (*A good book for those with no exposure to PDE to build up experience by working with concrete examples of PDE.*)

[T] M. E. Taylor *Partial Differential Equations, Basic Theory*, Springer, (1996). ISBN 0-387-94654-3.

Syllabus (Tentative)

I - The ODE Theorem in $\mathbf{R}^{\mathbf{n}}$ and Banach Spaces

II - Introduction to Fourier Analysis and Distributions

- II-1 The Schwartz Space $\mathcal{S}(\mathbf{R}^{\mathbf{n}})$
- II-2 The Fourier Transform on $\mathcal{S}(\mathbf{R}^{\mathbf{n}})$
- II-3 The Space of Temperate Distributions $\mathcal{S}'(\mathbf{R}^{\mathbf{n}})$
- II-4 The Space $\mathcal{E}'(X)$
- II-5 Test Functions and Convolutions
- II-6 Distributions on Open Sets
- II-7 Operations on Distributions
- II-8 Fundamental Solutions
- II-9 Paley-Wiener-Schwartz Theorem

III - Sobolev Spaces

- III-1 Definition and Basic Properties
- III-2 Sobolev's Imbedding Theorem and Algebra Property

IV - Laplace, Heat and Wave Equations (Fundamental Solutions)

V - The Linear Schrödinger and Korteweg-de Vries Equations (Cauchy Problem)

VI - Well-posedness of the Burgers Equation in Sobolev Spaces (Basic results)

VII - Well-posedness of the KdV Equation in Sobolev Spaces (Basic results)

VIII - Well-posedness of the NLS Equation in Sobolev Spaces (Basic results)

IX - Initial-Boundary Value Problems (The Fokas approach)

X - The Cauchy-Kovalevski Theorem