Math 80750: Topics in Differential Geometry

Spring 2024

Instructor: Kai-Wei Zhao, 118 Hayes-Healy Bldg, kzhao4@nd.edu
Class Time: MW 2–3:15pm

Descriptions: This course will present geometric problems in General Relativity. In the first part of the course, we will introduce the background knowledge of General Relativity including the basics of Lorentz geometry, the Einstein equations, matter fields and stress-energy tensor, energy conditions, static and stationary solutions, Cauchy problem, initial data and constraint equations, trapped surfaces, asymptotically flat spacetime, and gravitational energy-momentum.

In the second part of the course, we will present the Penrose Singularity Theorem, the Positive Mass Theorem, and the Density Theorem for the constraint equations. The tools used to tackle these problems involve various topics in geometric analysis, including minimal hypersurfaces, conformal geometry, and the elliptic theory on weighted Sobolev and Hölder spaces. The plan may change in the future.

Prerequisite: Riemannian geometry and basic knowledge of elliptic linear partial differential equations.

References:

1. "General Relativity", by Robert M. Wald
2. "Geometric Relativity", by Dan A. Lee
3. "Semi-Riemannian Geometry with Applications to Relativity", by Barrett O'Neill
4. "The Large Scale Structure of Space-Time", by S.W. Hawking and G.F.R. Ellis