<u>MATH 80520 – Topics In Logic – Spring 2024</u>

Instructor: Professor Dobrinen ndobrine@nd.edu 272 Hurley

Gödel's Incompleteness Theorem proved that any axiom system which is computable and strong enough in which to do ordinary number theory must necessarily be incomplete: there are statements which are neither provable nor disprovable from the axioms. Forcing is one method for building models of axioms which can answer questions which are not decidable by the axioms themselves. A famous example is Cantor's Continuum Hypothesis: "Every infinite set of real numbers is either countable or else in one-to-one correspondence with the set of all real numbers." Gödel showed CH to be consistent with ZFC by building the universe of construcible sets. On the other hand, Cohen showed the negation of CH to be consistent with ZFC, developing the method of forcing for his proof. It is this method and its modern extensions that will be the focus of this course.

One branch of modern set theory uses forcing to build models of ZFC to decide questions regarding infinite structures, including in analysis, topology, and algebra. Basically, any infinite structure which is not "definable" in some sense (e.g., Borel or analytic) is subject to independence phenomena, and forcing is a staple of these investigations. This course will cover the basics of forcing, product forcing, and iterated forcing. The first half of the course will delve into Chapters 4 and 5 of Kunen's book, *Set Theory*. The second half of the course will focus on the interactions between forcing and Ramsey theory, building up to Ramsey theorems on relational structures and topological Ramsey space theory. Knowledge of basic set theory, including ordinal and cardinal arithemetic and some understanding of L and Martin's Axiom will be assumed.

Reference books include *Combinatorial Set Theory: With a gentle introduction to forcing*, by Halbeisen; *Set Theory*, by Jech; *The Higher Infinite*, by Kanamori; and *Introduction to Ramsey Spaces*, by Todorcevic.

<u>Grading</u>

This will be run as a reading course with two somewhat disjoint intentions: (1) preparing students for research in set theory and (2) building general knowledge for all graduate students in logic and related areas. Students aiming to do research in set theory will be required to turn in and present homework problems. All students will be expected to give presentations of various material throughout the course. Group work is encouraged.