

MATH 80440: Topics in the braid group
Spring 2024

Meeting Time: MW 3:30–4:45 PM

Location: Hayes Healy 125

Instructor: Nick Salter

Office: Hurley 277

Email: nsalter@nd.edu

Office hours: Location: Hurley 277

TBD

Course Website: <https://nsalter.science.nd.edu/teaching/braidsspring2024/>

Textbook: For the first couple of weeks we will follow Chapter 9 of *A Primer on Mapping Class Groups* by Farb-Margalit. We will then draw mostly on the primary literature; a detailed list of sources will be maintained on the course webpage.

Plan for the course: We will begin with the basic theory of the braid group as both the fundamental group of a configuration space and as a mapping class group, and establish basic structural properties following the treatment in Farb-Margalit. The remainder of the course will be a sequence of largely-independent modules. Background will be presented as necessary. Some possibilities are listed below, but I also welcome suggestions based on the interest of the students.

- **Braid groups and knot theory:** Representability of knots and links as braid closures. Knot invariants (Alexander, Jones polynomials) from braid group representations.
- **Braid groups as mapping class groups:** Representations via the mapping class group perspective. Structural properties (linearity, super-rigidity, congruence subgroups).
- **Braids and polynomials:** Solubility and insolubility of polynomials. The work of Smale and McMullen on algorithmic complexity. Braid groups and Hodge theory. Quasipositive braids and links of algebraic functions.
- **Braids in geometric group theory:** (dual) Garside structures; the work of Birman-Ko-Lee. Work of Brady and McCammond on $K(\pi, 1)$ spaces for braid groups. Introduction to Artin groups.

Here is a day-to-day plan (subject to revision!).